Examining the Development of K-12 Students’ Cognitive Presence over Time: The Case of Online Mathematics Tutoring

Stefan Hrastinski
Stefan Stenbom
KTH Royal Institute of Technology, Sweden

Mohammed Saqr
University of Eastern Finland, Finland

Malin Jansson
Olga Viberg
KTH Royal Institute of Technology, Sweden

Abstract
In this article, we focus on the cognitive presence element of the Community of Inquiry (CoI) framework. Cognitive presence consists of four categories: Triggering Event, Exploration, Integration, and Resolution. These categories have been described as phases following an idealized logical sequence, although the phases should not be seen as immutable. Few studies have empirically examined how the four categories develop over time during the inquiry process. This article uses learning analytics methods to study transitions between the categories in K-12 online mathematics tutoring. It was statistically most probable that the tutoring sessions started with Triggering Event (95%) and then transitioned to Exploration (51%). The transitions from Exploration to Integration (18%) and Integration to Resolution (21%) achieved statistical significance but were less likely. In fact, it was more likely that the tutoring sessions transitioned from Integration to Exploration (39%) and Resolution to Exploration (36%). In conclusion, the findings suggest that the idealized logical sequence is evident in the data but that other transitions occur as well; especially Exploration recurs throughout the sessions. It seems challenging for students to reach the Integration and Resolution categories. As the CoI framework is commonly adopted in practice, it is important that tutors and educators understand that the categories of cognitive presence will often not play out in idealized ways, underlining their role in supporting how the inquiry process unfolds. In order to gain an improved understanding of the inquiry process, future research is suggested to investigate how the presences and categories of the CoI framework develop over time in different educational settings.

Keywords: Cognitive presence, community of inquiry, time, online mathematics tutoring

Hrastinski, S., Stenbom, S., Saqr, M., Jansson, M., & Viberg, O. (2023). Examining the development of K-12 students’ cognitive presence over time: The case of online mathematics tutoring. *Online Learning, 27*(3), 252-271. DOI: 10.24059/olj.v27i3.3481

The Community of Inquiry (CoI) theoretical framework (Garrison et al., 1999) is one of the most well-researched models for online and blended learning (Bozkurt & Zawacki-Richter, 2021; Castellanos-Reyes, 2020; Park & Shea, 2020). It assumes that learning is an active process where students construct and confirm meaning guided by practical inquiry. The basic structure of the CoI framework consists of three elements: the social, cognitive, and teaching presences. *Social* presence reflects the human experience of learning; *cognitive* presence outlines a constructivist learning process, and *teaching* presence is the organization and guidance required to promote learning (Garrison et al., 1999). The elements are divided into different categories representing their distinctive aspects. The elements and categories are regarded as both independent as they represent specific characteristics of a learning experience and overlapping as the intersection of the constructs enable interaction and progression of the inquiry (Arbaugh et al., 2008).

Various studies have explored relationships among the CoI’s presences. Through structural equation modeling, Shea and Bidjerano (2009) found that 70% of the variance in students’ levels of cognitive presence can be modeled based on their perceptions of their instructors’ skills and their abilities to establish a sense of social presence. Kozan and Richardson (2014) explored the relationships between and among teaching, social, and cognitive presences. Their result confirmed positive relationships between the elements. The authors also found that cognitive presence may impact the teaching presence–social presence relationship. Garrison et al. (2010) detected causal relationships among the presences. That is, teaching and social presences influenced cognitive presence, and teaching presence was found to influence social presence. Gutiérrez-Santiuste and Gallego-Arrufat (2017) examined the co-occurrence of elements and categories in chat, forum, and email in an information and communication technology course. They found that social presence interacts with the other elements.

While the CoI framework describes elements and categories that characterize inquiry processes, there is a limited understanding of how the different elements and categories develop *over time*. This is important since this can give us a more detailed understanding of the inquiry process and how to improve teaching methods. One exception is Akyol and Garrison (2008), who explored the dynamics of an online graduate course. Transcript analysis using the CoI framework coding scheme was used to investigate how the elements and categories of the CoI framework change over time. The nine-week course discussions were divided into the first, middle, and last three weeks in order to form measure points that were tested using ANOVA, with repeated measures for the categories. Their findings indicated significant changes in social and teaching presence categories over time. For social presence, affective expression, indicated by self-projection and expressing emotions, was reduced, while group cohesion, indicated by group identity and collaboration, increased significantly over time. For teaching presence, the category of direct instruction increased significantly during the course. However, for the cognitive presence categories, there were no statistically significant changes over time.

In this paper, we seek to raise awareness of the evolution of cognitive presence over time through educational interaction. The categories of cognitive presence—triggering event, exploration, integration, and. resolution—correspond to the different phases of thinking and learning in a Community of Inquiry. Cognitive presence also has a unique feature in relation to the other elements as Garrison et al. (2001) argue that the categories are, in fact, four phases and note that they are an “idealized logical sequence” (p. 9) of the inquiry process. At the same time, new triggering events can be introduced in a conversation, causing the process to start over.

Moreover, intuitive leaps shortcutting the logical inquiry phase may occur (Garrison & Archer, 2000). This paper demonstrates a quantitative evaluation of the idealized logical sequence of a cognitive presence.

Cognitive presence is central to the mathematical problem-solving process (Mills, 2016). Online mathematics tutoring has been found to be an effective and flexible way to support student learning in classrooms (Bloom, 1984; Wood et al., 1976) and online settings (Chappell et al., 2015; Tsuei, 2017). While most previous research has adopted a tutor perspective, for example, by focusing on the examination of the tutoring process and how to encourage collaborative learning in groups (e.g., McPherson & Nunes, 2009; Salmon, 2000), this study adopts a student perspective and pays special attention to how students develop *cognitive* presence over time in tutoring sessions. The aim of this paper is to investigate how students' cognitive presence develops over time in K–12 online mathematics tutoring. In addressing this aim, transcripts of online mathematics tutoring have been coded and analyzed by using learning analytics methods. More specifically, we address the following questions:

1. To what degree do the categories of cognitive presence follow an idealized logical sequence?
2. How do the categories of cognitive presence develop over time in online tutoring sessions?

Literature Review

In this section, we discuss research on cognitive presence, online tutoring and learning analytics, and how these relate to each other.

Cognitive Presence

Cognitive presence is an operationalization of Dewey's (1933) practical inquiry, defined as "the extent to which learners are able to construct and confirm meaning through sustained reflection and discourse" (Garrison et al., 2001, p. 11). It is suggested to follow four phases of the practical inquiry model: a triggering event, exploration, integration, and resolution (Garrison et al., 2001). A triggering event is the identification, conceptualization, and formulation of a problem or issue or when a conversation changes direction. The triggering event is logically the reason why a student initiates a tutoring conversation or draws attention to new problems or issues that arise during the tutoring session. Exploration includes reviewing the student's previous knowledge, brainstorming, and exchanging information. It might also include self-questioning and doubt on one's ability. Integration is about combining thoughts and making them operational. A typical example in online mathematics tutoring is the use of calculations (Stenbom et al., 2016). Resolution is about solving a problem or issue, which includes developing and analyzing potential solutions.

Online Tutoring

Tutoring is defined as "the means whereby an adult or 'expert' helps somebody who is less adult or less expert" (Wood et al., 1976, p. 89). Student learning is supported by interacting with a more skilled tutor (McPherson & Nunes, 2009). Bloom (1984) studied individual tutoring and compared it to a conventional control class. He found that tutored students were above 98% of the students in the control class when comparing final achievement measures. More recent research has found that online tutoring is also effective. In a study on 119 struggling students, it was found that online synchronous tutoring contributed to improvement in student assessment scores and mainly positive student perceptions (Chappell et al., 2015).

Turula (2018) showed that the levels of cognitive presence were strong in online tutorials and students reached higher levels of critical thinking than in face-to-face meetings (which supported social presence better). It was argued that social presence paved the way for cognitive presence. In a systematic review, it was found that most student contributions were categorized as exploration and integration, while triggering event and resolution occurred less frequently (Sadaf et al., 2021). However, the results are conflicting. In a study of a peer-facilitated discussion environment, cognitive presence was detected in most messages, although most student contributions were categorized as Triggering Event and Exploration (Chen et al., 2019). However, when tutors were involved, the frequency of integration and resolution increased significantly, emphasizing the importance of the tutor. On the contrary, Mills (2016) found that many students reached resolution early but were often not cognitively present in follow-up posts when students were asked to defend their solutions. Others investigated how student online discussions with high levels of cognitive presence can be designed (Gašević et al., 2015). They found that using participation guidelines combined with grading decreased the number of posts characterized as triggering event and exploration and increased the number of posts characterized as integration and resolution. Galikyan and Admiraal (2019) also studied online discussions. They found that engagement in integration and resolution predicted academic performance. Although triggering event and exploration are essential in online tutoring, these findings indicate the importance of supporting students in also achieving integration and resolution.

Learning Analytics

In this study, we use learning analytics (LA) methods to analyze how the four categories of cognitive presence develop over time. LA has been argued to offer valuable methods to increase our understanding of cognitive presence and the CoI framework, especially for students' knowledge construction (Kovanović et al., 2015), and to investigate temporal aspects of the learning process in computer-supported collaborative learning settings (Lämsä et al., 2021). Through the use of LA methods, scholars have been able to identify students' profiles in a study of online discussions. These profiles were characterized as 1) task-focused users, 2) content-focused no-users, 3) no-users, 4) highly intensive users, 5) content-focused intensive users, and 6) socially-focused intensive users (Kovanovic et al., 2015). Task-focused users were as successful as more intensive students belonging to profiles four to six, indicating that cognitive presence can be developed in different ways and is not necessarily connected to how frequently students contribute to online discussions. In another study, Kovanović et al. (2018) observed much smaller differences in cognitive presence when comparing passive users, task-focused users, and highly active users in a MOOC. It was hypothesized that this was likely the result of using a self-reported survey instrument, which has a self-selection bias. Thus, the use of content analysis (used in the present study) seems to be a methodological strength.

Taking a different approach, Yılmaz (2020) investigated the effect of providing LA-based feedback on the perceptions of cognitive, teaching, and social presence. He found a statistically significant effect on the three types of presences, underlining the importance of providing feedback to encourage the development of CoI and that such feedback can be automated. It can be noted that most previous research on LA and cognitive presence has used automated methods (Kovanović et al., 2015; Kovanović et al. 2018; Yılmaz, 2020). Recently, researchers have also used LA methods to examine the cognitive presence dimension (among others) of the quality and depth of student participation in online discussion forums (Farrow et al., 2021). The obvious benefit is to analyze large data sets. Our research complements this approach by using LA methods to analyze manual transcript analysis, with the benefit of providing a more detailed and rigorous analysis.

In this study, we use two LA methods: sequence and process mining. Sequence mining is frequently combined with process mining. It is an analytical technique that has been implemented frequently in LA research to capture the sequential ordered patterns of students' activities. Sequence mining has been used to analyze learning activities visually and statistically. The method has been used to detect types of learning tactics and their sequences. For instance, Matcha et al. (2019) used sequence mining to discover subgroups within learning actions and later correlate such subgroups to performance. Another recent example is the work of López-Pernas et al. (2021), who used sequence mining to discover the process by which students learned programming, how they succeeded in solving assignments, and when they struggled with their learning.

Process mining is a method that allows researchers to make sense of temporal data by discovering the process and mapping it visually and statistically. In doing so, process mining offers a summarizing, visually intuitive map of how students, e.g., use a learning tactic, move between tactics, and the time-frequency of such action. Since a learning process is a temporal process that unfolds over time (Reimann, 2009), the method has been used by many researchers to understand the learning process. For instance, Matcha et al. (2019) analyzed how students used different strategies and how efficient strategies were related to better performance and feedback. Sedrakyan et al. (2016) used process mining to understand students' complex problem-solving processes and offer relative feedback. Another recent example is the work of Peeters et al. (2020), who examined how students used self-regulated learning tactics in an online discussion about academic writing and how the different tactics were used by high and low achievers.

Method

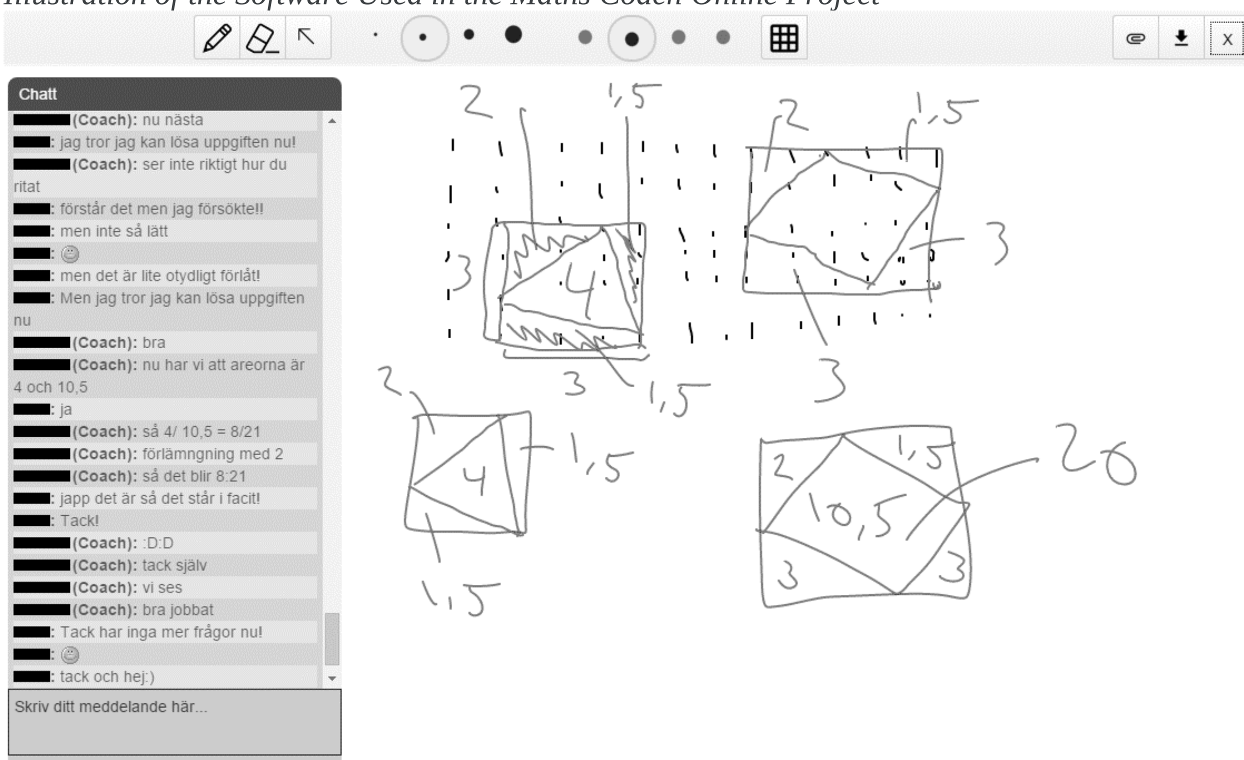
To address the research questions, a case study research design was selected. Using the CoI transcript coding procedure, math tutoring conversations were coded into the categories of cognitive presence: triggering event, exploration, integration, and resolution. In order to analyze how the categories develop over time, sequence and frequency mining were used.

Case Study Setting

The research design is a case study. Case studies allow an in-depth and detailed analysis of a phenomenon within a bounded context (Merriam, 1988). The Maths Coach Online project can be considered an ideal case to evaluate the idealized logical sequence of cognitive presence, as problem-solving is in focus and conversations consist of triggering events, exploration, integration, and resolution in the same chat (Stenbom et al., 2016). Therefore, the empirical data for this research was collected from the Maths Coach Online project, which was started in 2009. It offers K-12 students help with their homework in mathematics from online tutors. The tutors are teacher-students of mathematics and work evenings Monday to Thursday. The project includes three Swedish universities and one UK university. The tutors attend a 2 ECTS credit course to prepare them for online tutoring in mathematics. They use software specifically developed for the Maths Coach Online project that includes a queuing system, text-based chat, and digital whiteboard (see Figure 1). The use of chat makes it possible for the tutor to work with several students simultaneously (Chappell et al., 2015; Madden & Slavin, 2017). A benefit for students is that they can take time to work independently on a problem and continue the conversation later. The Anonymous project meets several research-based recommendations for online mathematics instruction, such as tutor professional development, office hours, frequent communication, and tailored advising (Coleman et al., 2017), though it should be regarded as a complement to conventional K-12 education.

Figure 1

Illustration of the Software Used in the Maths Coach Online Project



Data Collection

All tutoring conversations of the Maths Coach Online project are stored in a database. For this article, all conversations from one year of service (7,640 conversations) were made available for research. In the conversations, K-12 student were seeking guidance with their math homework covering all parts of the Swedish math curriculum (i.e., understanding and use of numbers, algebra, geometry, relationships, and statistics). The number of conversations selected was 60, with 3,109 messages sent in total, as it was thought to be a reasonable number for manual coding. We selected students who represented different age groups because it might affect the characteristics of the conversations. The students were between 12 and 19 years old. Half of the conversations were selected randomly (10 per educational stage of the students, up to age 12, ages 13 to 16, and ages 16 to 19). The additional 30 conversations were randomly selected from the entire dataset. Prior to using the Maths Coach Online service, students and guardians gave their informed consent that the tutoring sessions can be analyzed for research purposes. The tutoring sessions were anonymized prior to analysis.

Data Coding

The data coding was conducted using the validated CoI transcript coding procedure. With CoI, transcripts are a commonly used data source for analyzing discourse from educational activities (e.g., discussion forums, meetings, or chats). In fact, the CoI was originally developed by Garrison et al. (1999) to guide computer-conferencing transcript analysis. With this method, a coding scheme and a unit of analysis are defined, and transcripts are investigated for the elements and categories of CoI (Garrison et al., 2006). The method has been widely employed in different contexts (Weltzer-Ward, 2011; Kovanović et al., 2015; Kineshanko, 2016; Lee et al., 2022) but has also been criticized for not fully covering all aspects of critical thinking (Breivik, 2016). This study uses a cognitive presence coding scheme adapted to online mathematics tutoring (see Table 1). The slight adjustment from Garrison et al. (2001) was made to ensure that? some indicators reflect the one-to-one environment (instead of an environment with several students) and to demonstrate the focus on problem-solving. Additionally, math tutoring examples were provided. A key question when coding data is to select the unit of analysis (Garrison et al., 2006). Examples include thematic units, sentences, paragraphs, and messages (De Wever et al., 2006). During initial coding, it was observed that each chat message typically serves a specific purpose to move the conversation forward. Thus, each chat message was used as a unit of analysis.

Table 1

Coding of Cognitive Presence in Online Mathematics Tutoring (Stenbom et al., 2016).

Element	Category	Indicators (examples only)	Example
Cognitive presence	Triggering Event	Stating a problem	"Here's the problem …"
		Changing direction	"I have another issue."
	Exploration	Brainstorming	"Perhaps I could use …"
		Broad search for insight	"Am I thinking right here?"
		Information exchange	"What is a square root?"
	Integration	Connecting ideas	"I can combine … and …"
		Computations	"7/2 – x = 1/4 "
	Resolution	Achieve solution	"The answer is 3!"
		Analysis of solution	"I made a mistake with …"
		Implementation	"Then the apple is cheaper..."

The coding of the data was performed by one of the authors and a master's student. First, some conversations were examined in order to discuss how to interpret the coding scheme. Then, half of the conversations were coded independently by each person. A message could include more than one category. In these cases, the included categories were ordered in a sequence, as they occurred. Finally, ten conversations were coded by both persons in order to calculate inter-rater reliability. For transcript coding using the CoI framework, percent agreement is a recognized reliability measurement (Cohen, 1960; De Wever et al., 2006; Garrison et al., 2006). The percent agreement was .79 and Cohen's kappa for agreement beyond chance was .69 which indicates substantial agreement. In total, 1 042 messages were coded as cognitive presence. Table 2 presents the distribution among categories.

Table 2

Codes for Cognitive Presence

Category	**Number of codes**
Triggering event	150
Exploration	569
Integration	198
Resolution	125

Data Analysis

To investigate how the four categories of cognitive presence develop over time, the authors performed sequence mining. The sequence mining process entails using time-ordered sequences which are grouped within a time period or session. The sequences in our study were coded messages arranged according to their corresponding timestamp, while the conversations (full thread of messages) were grouped as sessions. To apply a process-oriented analysis to the data, we used the sequence and process mining methods that have been established in educational research in analysis of the temporal unfolding of time-stamped data (see section 2.3 for examples). Sequence mining uncovers the temporal unfolding of events (coded CoI categories in our case) and process mining shows the transition patterns e.g., how students transition from Exploration to Integration and at which frequency or proportion. Therefore, both methods are necessary to understand the different temporal patterns and offer a holistic view of the process. The timely ordered four categories of data were used to construct a state sequence object using the Traminer package (Gabadinho et al., 2011). To visually show the trajectory of messages, the sequences were plotted using an index plot, where every conversation is represented as a single trajectory formed of stacked bars. The stacked bars are the sequences of messages colored according to their coded category. A distribution plot was also plotted as the distribution of sequences at each time point.

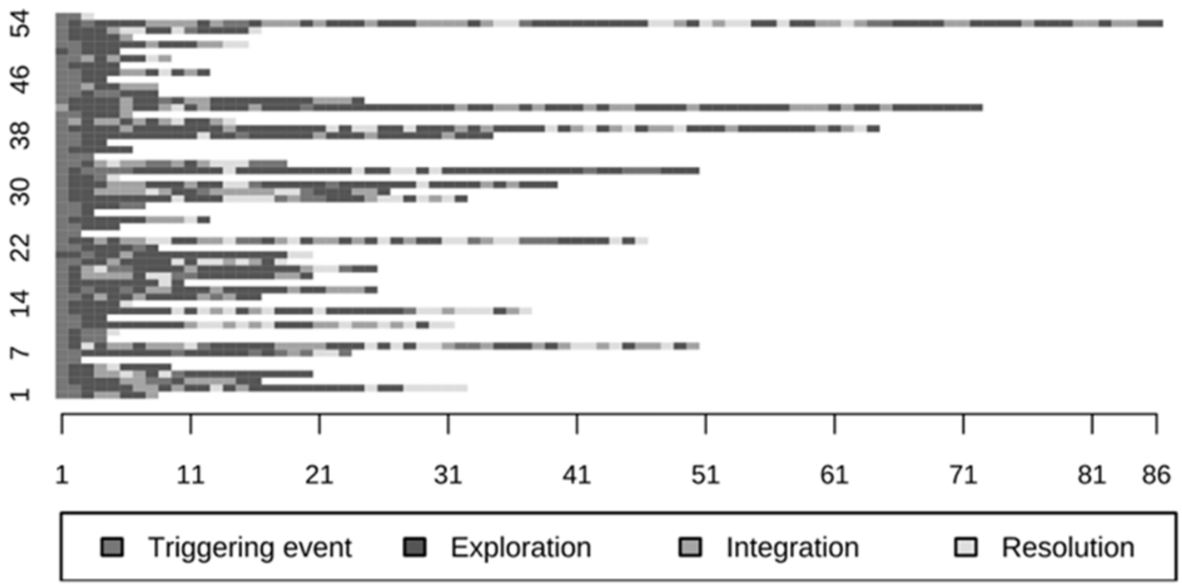
To understand how the categories develop over time, two types of process mining were performed: 1) Frequency-based process mining using the Bupar R! Package (Janssenswillen et al., 2019) and 2) First Order Markov Model, which shows the transitional probabilities between categories (Gatta et al., 2017). Firstly, relative case frequency-based process maps were constructed to show how students write messages coded as different categories and how they transition between them. The sequence maps were built by using each coded message as "event," the timestamp of the event as the timing, the user's ID as the case ID. The process map was the relative case frequency, i.e., the fraction of students writing a message coded as, e.g., Exploration, and the edges are the fraction of students transitioning to other categories, e.g., Integration. Secondly, stochastic process analysis was performed with the R library PMineR (Gatta et al., 2017). In contrast to the frequency-based algorithm, PMineR offers process visualization based on First Order Markov Models (FOMMs) with transition probabilities. In simple words, the process computes the probability of transition between events with statistical significance and only the statistically significant edges are plotted. The process plotted with FOMM is based on the fraction of coded messages, e.g., the fraction of messages coded as Exploration that would transition to Integration.

Results

Figure 2 shows an index plot where the categories are represented as greyscale bars for each tutoring session. It is evident that most sessions start with Triggering events followed by Exploration. Then, Exploration, followed by Integration, are the most common categories throughout the session. There are six sessions with 45 messages or more, all of which focus on Exploration and Integration during the final part of the session rather than Resolution.

Figure 2

Index Plot of the Sequence of the Categories in Each Conversation



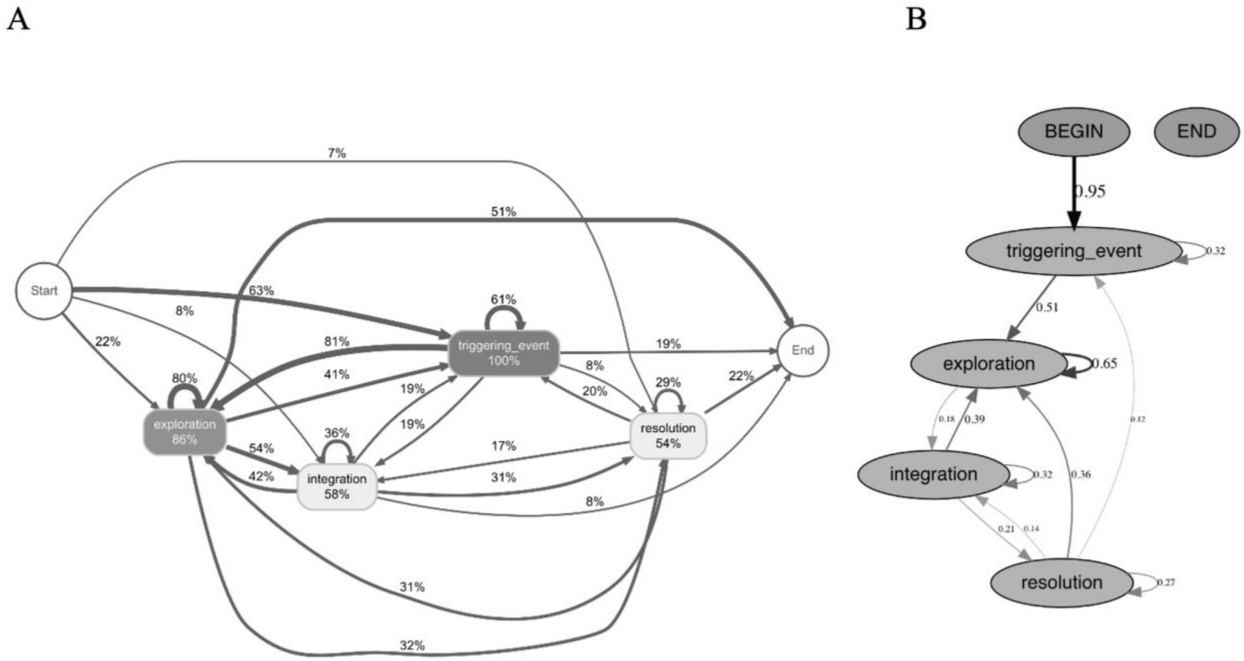
Note. The Y-axis represents the number of conversations; X-axis represents the order of interactions.

Two types of process mining were implemented: a relative case-based algorithm that shows the students’ transition between the categories (see Figure 3A) and a stochastic process map (see Figure 3B) based on the fraction of messages. As evident in Figure 3A, a majority of students (63%) start with a Triggering event, although always express Triggering even at some point (100%). This is followed by the transition from Triggering event to Exploration (81%), which is used at least once in most conversations (86%). Around half of the students transition from Exploration to Integration (54%) to end the conversation (51%) or move back to a Triggering Event (41%). Integration is displayed in 58% of the students’ conversations. The most common students’ transition from Integration is back to Exploration (42%) and to Resolution (31%). Resolution is the least frequent category displayed in 54% of the students’ conversations. The most common student’s transition from Resolution is back to Exploration (31%). It is also notable that many students’ conversations include Exploration followed by another message of Exploration (80%) and a Triggering Event with another message of Triggering event (61%).

The stochastic process map is based on transitional probabilities computed based on the FOMM. Here the transitions describe the events (compared to students in the previous map). Figure 3B shows the most probable *first transition* from a category to another category. The probability that a tutoring session starts with a Triggering Event is 95%. The most probable first transition from Triggering Event is to Exploration (51%). Then, the most likely transition is from Exploration to Exploration again (65%), and less likely to Integration (18%). The next most likely transition is from Integration back to Exploration (39%) or to Resolution (21%). There was no statistically significant transition to the end of the conversation, meaning that the tutoring sessions can end with different categories, although rarely with a Triggering Event.

Figure 3

Frequency-Based Algorithm (A) and Stochastic Process Map (B)



In Table 3, all possible transitions between the categories of Cognitive presence, the statistical probability according to the FOMM algorithm and an example from the conversation logs for each transition is presented. Notably, transitions could occur between all the categories, and within the categories. That said, as noted above, the transitions that achieved the strongest statistical significance were Exploration to another message of Exploration (65%), Triggering Event to Exploration (51%), Integration to Exploration (39%), Resolution to Exploration (36%), Triggering Event to another message of Triggering Event (32%) and Integration to another message of Integration (32%), and Resolution to another message of Resolution (27%).

Table 3
Transitions, Probabilities, and Examples

From	To	Probability (FOMM)	Example
Triggering Event	Triggering Event	0.32	Triggering Event: I need help with a couple of math assignments Triggering Event: 21a) Calculate the volume of the balcony box 60 * 20 * 15
	Exploration	0.51	Triggering Event: What is the largest 90 cm or 893 mm? Exploration: so, it goes 10 mm on 1 cm?
	Integration	-	Triggering Event: and so I need help with division so if we take 146,173,146,135 as number a should calculate the median value how should I divide then? Integration: I know you have to add everything and it will be 600 a divided by 4
	Resolution	-	Triggering Event: (-1) +4 Resolution: 3 apples
Exploration	Triggering event	-	Exploration: so it should be x=-2 +- the root from 2^2+0 which then becomes x = -2+. The root from 4 +-* Triggering Event: 2 (x-2)(x + 4) = (x-3)^2 what should I use for method then ??
	Exploration	0.65	Exploration: then I do 0.9991 ^ (1/19) = 0 !! ??? Exploration: but the answer is 9.23
	Integration	0.18	Exploration: Why did I make a mistake? Integration: I think that 2a has a plus sign right next to it, so it should be added with 2a and then subtracted with 4f 2a + 2a - 4f
	Resolution	-	Exploration: yes. Resolution: or 47!
Integration	Triggering event	-	Integration: 2,5,9 Triggering Event: and so I need help with division so if we take 146,173,146,135 as a number a should calculate the median value how should I divide then?
	Exploration	0.39	Integration: 1a + 4r? Exploration: aa right. Can I have a slightly more difficult problem, to see if I can handle it then?
	Integration	0.32	Integration: is it maybe 2.2 cm then? Integration: 89, 3cm?

	Resolution	0.21	Integration: - 4x it should be Resolution: ok now I got it right so x = 6
Resolution	Triggering Event	0.12	Resolution: this I do not need pq for Triggering event: hi Coach do you want to help me a little with a problem??
	Exploration	0.36	Resolution: The answer is: 1301.5 Exploration: Aha now I understand What to multiply with
	Integration	0.14	Resolution: 18000 liters Integration: aha you should divide it by 1000 so it will be 18 liters
	Resolution	0.27	Resolution: 6 if you round off Resolution: got the answer 5.6569

Discussion

The foundational work on cognitive presence and CoI argued that the four categories of cognitive presence are, in fact, four phases in the Practical Inquiry model (Garrison et al., 2001). The categories were described as an idealized logical sequence of cognitive presence, although the categories were argued not to be regarded as immutable. New triggering events can be introduced in a conversation, causing the process to start over (Garrison et al., 2001), and sometimes phases are skipped, and conceptual leaps are made (Garrison & Archer, 2000). However, while these categories have been used and tested in many empirical studies, few have investigated how the categories develop over time. In doing this, one of the key theoretical claims of cognitive presence is empirically tested. This article adopted LA methods to analyze how the four categories developed over time in a setting of online mathematics tutoring. More specifically, the first research question asked to what degree the categories of cognitive presence followed an idealized logical sequence?

An index plot, frequency map, and stochastic process map, the latter computed based on the First Order Markov Models (FOMM) algorithm, were developed. It was most probable that tutoring sessions started with a Triggering Event (95%). This finding is not aligned with a systematic review of cognitive presence suggesting that most contribution were categorized as Exploration and Integration, rather than Triggering Event and Resolution (Sadaf et al., 2021). However, this can be explained by that the Maths Coach Online project offers K-12 students help with their homework in mathematics from online tutors during evenings. Students typically use the service when they need help with a certain mathematical problem. In line with the theoretical claim of the cognitive presence element (Garrison et al., 2001), it is probable that students, after describing the problem, transition to Exploration (51%), during which the tutor and student brainstorm and exchange information. It is common that students then continue to write messages coded as Exploration (65%). However, the probability that the remaining idealized logical phases of cognitive presence would occur was much lower. The probability that students transition from Exploration to Integration was 18% and from Integration to Resolution was 21%. This finding is aligned with previous research, which has found that it is often challenging for students to move beyond Triggering Event and Exploration in order to engage in Integration and

Resolution (Garrison et al., 1999; Galikyan & Admiraal, 2019; Gašević et al., 2015; Vaughan & Garrison, 2005). This is essential because engagement in Integration and Resolution has been found to predict academic performance (Galikyan & Admiraal, 2019). That said, some students that use the Maths Coach Online service might only need help to get started and might reach the Integration and Resolution independently after the tutoring session.

The second research question investigated how the categories of cognitive presence developed over time in tutoring sessions. The findings related to the first research question confirmed that the categories of cognitive presence followed the idealized logical sequence to some extent, especially for the first two categories (Triggering Event and Exploration). Interestingly, it was more likely that students transitioned from Integration to Exploration (39%) and from Resolution to Exploration (36%) rather than according to the theoretical sequence. After briefly describing the Triggering Event, it seemed that Exploration was often the center of attention for students during the tutoring sessions. Gašević et al. (2015) found that the use of participation guidelines combined with grading decreased the number of posts characterized as Triggering Event and Exploration and increased the number of posts characterized as Integration and Resolution. This suggests that there might be a need to develop improved teaching and tutoring practices in order to encourage students to engage in Integration and Resolution. It is also important to consider the effects of the subject discipline. In mathematics, it is clear whether a student is able to solve a problem or not, while in other subject disciplines, the act of reaching resolution might be more subjective.

The practical implications for teachers and tutors relate to how they can facilitate students’ practical inquiry during the four phases. According to the present study and several others (e.g., Garrison et al., 2001; Neto et al., 2018; Guo et al., 2021), the vast majority of dialogue is focused on exploration. To achieve a more even distribution of cognitive presence categories in dialogue, teachers and tutors should develop strategies for promoting students' transitions to and retention during Integration and Resolution. Furthermore, it is clear that the Practical Inquiry model and the idealized logical sequence of the categories of cognitive presence should not be seen purely in deterministic ways. As noted by Garrison et al. (1999), the categories should not be regarded as immutable: New Triggering Events can be introduced, categories are skipped and conceptual leaps are made (Garrison & Archer, 2000). In our setting, Triggering Events especially occurred in the beginning of a conversation, while the remaining conversations mainly centered around Exploration. It is important that practitioners understand that the Practical Inquiry Model describes an idealized sequence rather than a detailed account of how inquiry processes play out in practice. Tutors and educators could reflect on what kind of inquiry processes they are aspiring to achieve. In some cases, Exploration could be the focus of a conversation, while in other cases it might be desirable to achieve Integration and/or Resolution. A key challenge for tutors and educators seems to not only be how to support Exploration, but also how inquiry processes could enter the Integration and Resolution phases, while also being receptive to new Triggering Events that might support students to engage in further inquiry processes.

Limitations and Further Research

The present study involves examining cognitive presence sequencing and development over time as measured by transcript coding. The transcript coding method is described as a “technique to understand interaction patterns and the quality of the discourse in online communities of inquiry ... It is through the use of transcript analysis that educators can investigate beyond what students say they do to review what they actually do" (Garrison, et al. 2006, p. 8). That said, transcript analysis only involves the discourse of an inquiry process and not the individual's critical thinking. Garrison et al. (2001) describe that cognitive presence involves students' practical inquiry through individual critical thinking and shared discourse. Following this, the present study is limited to the shared discourse as documented in the transcripts, while students' individual inquiry, e.g., what they are thinking or doing that is not visible in the chat, is not analyzed. The individual sequence of cognitive presence may follow different patterns than written communication.

This article focused on online mathematics tutoring, in which a tutor helps a K-12 student with a homework problem during the evening. This is of course a very different setting as compared with the studies of online classes that often characterize CoI research. In the Maths Coach Online project, the community consists of many dyadic relationships between tutors and students. It is likely that the sequence of categories of cognitive presence will look different in different settings. We believe online mathematics tutoring is a suitable case because one student was consistently the center of attention during tutoring. The purpose is to help somebody who is less adult or less expert (Wood et al., 1976) to solve, and *most importantly*, understand how to solve a mathematical problem. A complete understanding of the immutable and dynamic aspects of cognitive presence will require studies in several empirical contexts (i.e., small and large communities, synchronous and asynchronous interactions, shorter and longer sessions, written and spoken communication).

In this study, we focused on investigating how the categories of cognitive presence developed over time. Although we present how the categories of cognitive presence develop over time in a specific education situation, complementary methods would be necessary to explain why these transitions occur. A key challenge for the future is to gain an improved understanding of the inquiry process, i.e., how the different elements and categories of the CoI framework interplay and develop over time in different education settings.

Declarations

The authors have no conflicts of interest to disclose.

There is no need for ethical approval according to the Swedish Ethical Review Act (SFS 2003:460). The research adheres to the good research practice guidelines from the Swedish Research Council (2017).

The paper is co-funded by the Academy of Finland for the project TOPEILA, Decision Number 350560 which was received by Mohammed Saqr.

References

Akyol, Z., & Garrison, D. R. (2008). The development of a community of inquiry over time in an online course: Understanding the progression and integration of social, cognitive, and teaching presence. *Journal of Asynchronous Learning Networks, 12,* 3-22. http://files.eric.ed.gov/fulltext/EJ837483.pdf

Arbaugh, J. B., Cleveland-Innes, M., Diaz, S. R., Garrison, D. R., Ice, P., Richardson, J. C., & Swan, K. P. (2008). Developing a community of inquiry instrument: Testing a measure of the community of inquiry framework using a multi-institutional sample. *The Internet and Higher Education, 11*(3–4), 133–136. https://doi.org/10.1016/j.iheduc.2008.06.003

Bloom, B. S. (1984). The 2 sigma problem: The search for methods of group instruction as effective as one-to-one tutoring. *Educational Researcher, 13,* 4 –16. https://doi.org/10.3102/0013189X013006004

Bozkurt, A., & Zawacki-Richter, O. (2021). Trends and patterns in distance education (2014–2019): A synthesis of scholarly publications and a visualization of the intellectual landscape. *The International Review of Research in Open and Distributed Learning*, *22*(2), 19–45. https://doi.org/10.19173/irrodl.v22i2.5381

Breivik, J. (2016). Critical thinking in online educational discussions measured as progress through inquiry phases: A discussion of the cognitive presence construct in the community of inquiry framework. *International Journal of E-Learning & Distance Education*, *31*(1), 1. https://www.proquest.com/openview/b86446f9706eb9a3f1daf5091477a5b4/1

Castellanos-Reyes, D. (2020). 20 years of the Community of Inquiry framework. *TechTrends, 64*(4), 557-560. https://doi.org/10.1007/s11528-020-00491-7

Chappell, S., Arnold, P., Nunnery, J., & Grant, M. (2015). An examination of an online tutoring program's impact on low-achieving middle school students' mathematics achievement. *Online Learning, 19*(5), 37–53. http://files.eric.ed.gov/fulltext/EJ1085790.pdf

Chen, Y., Lei, J., & Cheng, J. (2019). What if online students take on the responsibility: Students' cognitive presence and peer facilitation techniques. *Online Learning, 23*(1), 37-61. https://files.eric.ed.gov/fulltext/EJ1210949.pdf

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement, 20*(1), 37–46.

Coleman, S. L., Skidmore, S. T., & Martirosyan, N. M. (2017). A review of the literature on online developmental mathematics: Research-based recommendations for practice. The *Community College Enterprise, 23*(2), 9–26. https://www.proquest.com/openview/f9356284ed4c10b28cd5f996b5a70cb8/1

Dewey, J. (1933). *How we think*. Dover Publications.

De Wever, B., Schellens, T., Valcke, M., & Van Keer, H. (2006). Content analysis schemes to analyze transcripts of online asynchronous discussion groups: A review. *Computers & Education, 46*(1), 6–28. https://doi.org/10.1016/j.compedu.2005.04.005

Farrow, E., Moore, J., & Gasevic, D. (2021, April). *A network analytic approach to integrating multiple quality measures for asynchronous online discussions*. In LAK21: 11th International Learning Analytics and Knowledge Conference (pp. 248–258). https://doi.org/10.1145/3448139.3448163

Gabadinho, A., Ritschard, G., Mueller, N. S., & Studer, M. (2011). Analyzing and visualizing state sequences in R with TraMineR. *Journal of Statistical Software, 40*(4), 1-37. http://www.jstatsoft.org/v40/i04

Galikyan, I., & Admiraal, W. (2019). Students' engagement in asynchronous online discussion: The relationship between cognitive presence, learner prominence, and academic performance. *The Internet and Higher Education, 43*, 100692. https://doi.org/10.1016/j.iheduc.2019.100692

Garrison, D. R., Anderson, T., & Archer, W. (1999). Critical inquiry in a text-based environment: Computer conferencing in higher education. *The Internet and Higher Education, 2*(2-3), 87-105. https://doi.org/10.1016/S1096-7516(00)00016-6

Garrison, D. R., Anderson, T., & Archer, W. (2001). Critical thinking, cognitive presence, and computer conferencing in distance education. *American Journal of Distance Education, 15*(1), 7-23. https://doi.org/10.1080/08923640109527071

Garrison, D. R., & Archer, W. (2000). *A transactional perspective on teaching and learning: A framework for adult and higher education*. Pergamon.

Garrison, D. R., Cleveland-Innes, M., & Fung, T. S. (2010). Exploring causal relationships among teaching, cognitive and social presence: Student perceptions of the community of inquiry framework. *The Internet and Higher Education, 13*(1-2), 31-36. https://doi.org/10.1016/j.iheduc.2009.10.002

Garrison, D. R., Cleveland-Innes, M., Koole, M., & Kappelman, J. (2006). Revisiting methodological issues in transcript analysis: Negotiated coding and reliability. *The Internet and Higher Education, 9*(1), 1–8. https://doi.org/10.1016/j.iheduc.2005.11.001

Gašević, D., Adesope, O., Joksimović, S., & Kovanović, V. (2015). Externally-facilitated regulation scaffolding and role assignment to develop cognitive presence in asynchronous online discussions. *The Internet and Higher Education, 24*, 53-65. https://doi.org/10.1016/j.iheduc.2014.09.006

Gatta, R., Lenkowicz, J., Vallati, M., Rojas, E., Damiani, A., Sacchi, L., ... & Valentini, V. (2017, June). *pMineR: An innovative R library for performing process mining in medicine*. In Conference on Artificial Intelligence in Medicine in Europe (pp. 351-355). Springer, Cham. https://link.springer.com/chapter/10.1007/978-3-319-59758-4_42

Guo, P., Saab, N., Wu, L., & Admiraal, W. (2021). The Community of Inquiry perspective on students' social presence, cognitive presence, and academic performance in online project-based learning. *Computer Assisted Learning*, 37(5), 1479–1493. https://doi.org/10.1111/jcal.12586

Gutiérrez-Santiuste, E., & Gallego-Arrufat, M. J. (2017). Type and degree of co-occurrence of the educational communication in a community of inquiry. *Interactive Learning Environments, 25*(1), 62-71. https://doi.org/10.1080/10494820.2015.1114498

Janssenswillen, G., Depaire, B., Swennen, M., Jans, M., & Vanhoof, K. (2019). bupaR: Enabling reproducible business process analysis. *Knowledge-Based Systems, 163*, 927-930. https://doi.org/10.1016/j.knosys.2018.10.018

Kineshanko, M. K. (2016). *A thematic synthesis of community of inquiry research 2000 to 2014* [Doctoral dissertation, Athabasca University]. http://hdl.handle.net/10791/190

Kovanović, V., Gašević, D., Joksimović, S., Hatala, M., & Adesope, O. (2015). Analytics of communities of inquiry: Effects of learning technology use on cognitive presence in asynchronous online discussions. *The Internet and Higher Education, 27*, 74-89. https://doi.org/10.1016/j.iheduc.2015.06.002

Kovanović, V., Joksimović, S., Poquet, O., Hennis, T., Dawson, S., Gašević, D., ... & Siemens, G. (2017). *Understanding the relationship between technology use and cognitive presence in MOOCs*. In Proceedings of the Seventh International Learning Analytics & Knowledge Conference (pp. 582-583). https://doi.org/10.1145/3027385.3029471

Kozan, K., & Richardson, J. C. (2014). Interrelationships between and among social, teaching, and cognitive presence. *The Internet and Higher Education, 21*, 68-73. https://doi.org/10.1016/j.iheduc.2013.10.007

Lee, J., Soleimani, F., Irish, I., Hosmer, Iv, J., Yilmaz Soylu, M., Finkelberg, R., & Chatterjee, S.. (2022). Predicting cognitive presence in at-scale online learning: MOOC and for-credit online course environments. *Online Learning, 26*(1). https://doi.org/10.24059/olj.v26i1.3060

Madden, N. A., & Slavin, R. E. (2017). Evaluations of technology-assisted small-group tutoring for struggling readers. *Reading & Writing Quarterly, 33*(4), 327–334. https://doi.org/10.1080/10573569.2016.1255577

McPherson, M., & Nunes, M. B. (2013). The role of tutors as a fundamental component of online learning support. *Distance and E-Learning in Transition*, 235-246.

Merriam, S. B. (1988). *Case study research in education: A qualitative approach*. Jossey-Bass.

Mills, J. J. (2016). *A mixed methods approach to investigating cognitive load and cognitive presence in an online and face-to-face college algebra course*. [Doctoral dissertation, University of Kentucky]. https://uknowledge.uky.edu/edsc_etds/8/

Neto, V., Rolim, V., Ferreira, R., Kovanović, V., Gašević, D., Dueire Lins, R., & Lins, R.. (2018). Automated analysis of cognitive presence in online discussions written in Portuguese. In *Lecture Notes in Computer Science* (pp. 245–261). https://doi.org/10.1007/978-3-319-98572-5_19

Park, H., & Shea, P. (2020). A ten-year review of online learning research through co-citation analysis. *Online Learning, 24*(2). https://doi.org/10.24059/olj.v24i2.2001

Peeters, W., Saqr, M., & Viberg, O. (2020). Applying learning analytics to map students' self-regulated learning tactics in an academic writing course. In Proceedings of the 28th International Conference on Computers in Education, Asia-Pacific Society for Computers in Education.

Reimann, P. (2009). Time is precious: Variable- and event-centred approaches to process analysis in CSCL research, *International Journal of Computer-Supported Collaborative Learning, 3*, 239-257. http://dx.doi.org/10.1007/s11412-009-9070-z

Salmon, G. (2000). *E-moderating: the key to teaching and learning online*. Kogan Page.

Sedrakyan, G., De Weerdt, J., & Snoeck, M. (2016). Process-mining enabled feedback: "Tell me what I did wrong" vs. "Tell me how to do it right." *Computers in Human Behavior, 57*, 352-376. https://doi.org/10.1016/j.chb.2015.12.040

Sadaf, A., Wu, T., & Martin, F. (2021). Cognitive presence in online learning: A systematic review of empirical research from 2000 to 2019. *Computers and Education Open*, 100050. https://doi.org/10.1016/j.caeo.2021.100050

Shea, P., & Bidjerano, T. (2009). Community of inquiry as a theoretical framework to foster "epistemic engagement" and "cognitive presence" in online education. *Computers & Education*, *52*(3), 543-553. https://doi.org/10.1016/j.compedu.2008.10.007

Stenbom, S., Jansson, M., & Hulkko, A. (2016). Revising the Community of Inquiry Framework for the Analysis of One-To-One Online Learning Relationships. *The International Review of Research in Open and Distributed Learning*, *17*(3). https://doi.org/10.19173/irrodl.v17i3.2068

Swedish Research Council (2017). Good research practice. https://www.vr.se/download/18.5639980c162791bbfe697882/1555334908942/Good-Research-Practice_VR_2017.pdf

Tsuei, M. (2017). Learning behaviours of low-achieving children's mathematics learning in using of helping tools in a synchronous peer-tutoring system. *Interactive Learning Environments*, *25*(2), 147–161. https://doi.org/10.1080/10494820.2016.1276078

Turula, A. (2018). The shallows and the depths. Cognitive and social presence in blended tutoring. *Technology, Pedagogy and Education*, *27*(2), 233-250. https://doi.org/10.1080/1475939X.2017.1370388

Vaughan, N., & Garrison, D. R. (2005). Creating cognitive presence in a blended faculty development community. *The Internet and Higher Education*, *8*(1), 1–12. https://doi.org/10.1016/j.iheduc.2004.11.001

Weltzer-Ward, L. (2011). Content analysis coding schemes for online asynchronous discussion. *Campus-Wide Information Systems*, *28*(1), 56–74. https://doi.org/10.1108/10650741111097296

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, *17*, 89–100. https://doi.org/10.1111/j.1469-7610.1976.tb00381.x

Yılmaz, R. (2020). Enhancing community of inquiry and reflective thinking skills of undergraduates through using learning analytics-based process feedback. *Journal of Computer Assisted Learning*, *36*(6), 909-921. https://doi.org/10.1111/jcal.12449